

An Efficient Fast Walsh-Hadamard Transform Based OFDM-IM Scheme with Lower PAPR

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Abstract—Orthogonal frequency division multiplexing-index modulation (OFDM-IM) is an upcoming technology for B5G/6G wireless networks. It is considered a competitive candidate due to its advantages in communication efficiency and signal transmission spectral efficiency. However, as a novel multicarrier modulation technique, OFDM-IM still inherits the problem of high power amplifier (HPA) cost caused by the distortion from its enormous Peak-to-Average Power Ratio (PAPR) when passing through non-linear devices. In this study, we propose a novel scheme called Walsh-Hadamard Transformed OFDM-IM (WHT-OFDM-IM) to improve system performance and combat the issue of PAPR in multiple subcarrier and multiple activated subcarrier scenarios. The simulation results demonstrate that WHT can significantly enhance communication efficiency and effectively decrease HPA costs. WHT-OFDM-IM demonstrates superior performance in PAPR and BER compared to conventional OFDM-IM, and has great potential for future wireless networks.

Index Terms—OFDM-IM, WHT-OFDM-IM, PAPR, Signal Processing, Precoding

I. INTRODUCTION

The pursuit of higher spectrum efficiency and energy efficiency at a low cost is a persistent goal in wireless communication systems. Multi-carrier technology, through parallel transmission, enhances spectrum efficiency in limited spectrum resources by transforming high-speed serial information flows into low-speed parallel information flows, which are then transmitted in superposition. Orthogonal frequency division multiplexing (OFDM) is a widely used waveform in Multi-carrier technology and a representative of 4G LTE and 5G new radio (NR), particularly in enhanced mobile broadband (eMBB) data transmission in multi-carrier systems, due to its high spectrum efficiency [1].

However, the next generation of B5G/6G wireless networks requires higher spectrum efficiency and superior performance. To meet these demands, the concept of Index Modulation (IM) has gained attention in recent years [2]. IM can be mainly divided into frequency domain and spatial index modulation. In 2009, Abu.alhiga and Hass proposed spatial modulation for OFDM systems, leading to the development of OFDM technology based on index modulation (SIM-OFDM) [3]. Basar introduced OFDM with Index Modulation (OFDM-IM) in 2013, a more flexible and systematic approach that allows for dynamic subcarrier number control and reduces error propagation in SIM-OFDM [4]. Studies have shown that OFDM-IM provides lower bit error rates (BER), faster transmission

rates, and higher spectrum efficiency. Advances in OFDM-IM technology continue with ongoing improvement and refinement, such as the selective mapping (SLM) scheme for PAPR reduction proposed by Si-Yu Zhang and Behnam Shahrava [5], and ongoing progress in low-complexity efficient detection [6].

On the other hand, Walsh-Hadamard Transform (WHT) is widely used in signal processing and precoding communications [7]. OFDM systems suffer from a major drawback of high PAPR, which decreases signal performance and causes distortion. This distortion affects the power amplifier output and can cause saturation at the digital-to-analog converter. PAPR also distorts the signal constellation and requires a large power back-off for operation, leading to insufficient performance[8]. Overcoming PAPR is necessary for MIMO-OFDM systems. The first proposal of WHT-OFDM significantly improved BER performance in 2000 [9]. Furthermore, WHT-OFDM can significantly reduce the high PAPR and reduce HPA costs [10]. The application of WHT in Non-orthogonal Multiple Access (NOMA) also improves 5G communication performance [11]. This paper proposes a novel system, Walsh-Hadamard Transformed OFDM-IM (WHT-OFDM-IM), by introducing WHT into OFDM-IM. Results show that WHT-OFDM-IM significantly reduces BER, especially in scenarios with a large number of subcarriers and effectively combats high PAPR in multiple activated subcarrier scenarios.

In this paper, we address the issue of enormous PAPR in the multiple activated subcarriers scenarios in OFDM-IM systems and achieve the ambitious goal of enhancing performance of OFDM-IM. To achieve this, we propose a novel system scheme, known as WHT-OFDM-IM, to enhance the efficiency of communication. The main contributions of this paper are summarized as follows:

- We introduce the transceiver structure of WHT-OFDM-IM, which incorporates WHT into the OFDM-IM system. This structure involves adding WHT between the IFFT and cyclic prefix in the transmitter, and adding IWHT between the FFT and cyclic prefix in the receiver, by comparing the transceiver structure of OFDM with WHT-OFDM.
- We then evaluated the PAPR of the WHT-OFDM-IM under different activated subcarriers by comparing the complementary cumulative distribution function (CCDF) curve between OFDM-IM and WHT-

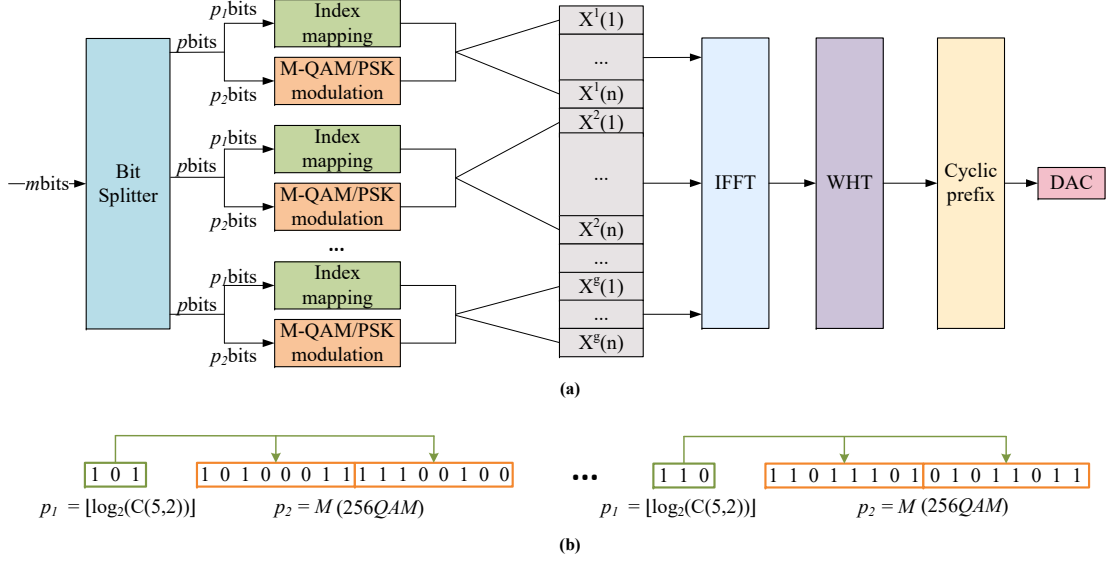


Fig. 1. Schematic illustration of the principle of WHT-OFDM-IM: (a) is the transmitter of WHT-OFDM-IM, and (b) is the example of the division between the index bit and information bit in OFDM-IM (The number of subcarriers is 5 and the activated one is 2 under 256QAM).

OFDM-IM.

- We also evaluated the BER performance of WHT-OFDM-IM by comparing it with OFDM-IM. Furthermore, we evaluated the impact on WHT-OFDM-IM under different number of subcarriers and activated subcarriers.

This paper is organized as follows: The WHT-OFDM-IM system model and the principle of Walsh-Hadamard Transform (WHT) are described in Section II. The comparison of the transceiver structure of WHT-OFDM-IM and OFDM-IM is presented in Section III, and the simulation parameters are determined in Section IV. The performance analysis of PAPR and BER of WHT-OFDM-IM and OFDM-IM is carried out in Section IV. Finally, the results are summarized and discussed in Section V.

II. WHT-OFDM-IM OPERATING PRINCIPLE

As a novel extension of conventional OFDM-IM, WHT-OFDM-IM offers lower PAPR and improved performance. Unlike conventional OFDM, WHT-OFDM-IM does not activate all subcarriers or transmit modulation symbols on all subcarriers. Instead, it selects a subset of subcarriers for activation and transmits modulation symbols on these active subcarriers. The transmitted information bits are divided into two parts: the first part is conveyed by the combination of activated subcarrier indices (referred to as index bits), and the second part is conveyed by the modulation symbols sent on the active subcarriers (referred to as symbol bits).

The transmitter [12] selects k activated subcarriers from the n subcarriers in the index mapper because the WHT-OFDM-IM subblocks are independent of each other. The combination of activated subcarriers for each WHT-OFDM-IM subblock β is shown as:

$$I_{\beta} = \{i_{\beta}^1, i_{\beta}^2, \dots, i_{\beta}^k\} \quad (1)$$

The transmitter of WHT-OFDM-IM is illustrated in Fig. 1(a). In each transmission block, the N subcarriers are divided into g groups, each group consisting of $n = N/g$ subcarriers. The binary bit sequence is also divided into g groups, with each group consisting of $p = m/g$ bits. The p bit sequence is separated into the index bit sequence p_1 and the information bit sequence p_2 . The number of index bits carrying information on the activated subcarriers in each WHT-OFDM-IM subblock transmission can be represented as:

$$p_1 = \frac{m_1}{g} = \lfloor \log_2 C(n, k) \rfloor \quad (2)$$

Similarly, the number of bits of modulation symbols carried by each WHT-OFDM-IM transmission subblock is:

$$p_2 = \frac{m_2}{g} = k(\log_2 M) \quad (3)$$

These p bits are then mapped to OFDM-IM subblocks. Finally, the g WHT-OFDM-IM subblocks are combined to form an OFDM-IM transmission block of length N [4].

$$m = pg = [k(\log_2 M) + \lfloor \log_2 C(n, k) \rfloor]g \quad (4)$$

As depicted in Fig. 1(b), the WHT-OFDM-IM index bit for each block under 256QAM is $p_1 = 3$ and the information bit is $p_2 = 8$ for $n = 5$ subcarriers and $k = 2$ activated subcarriers. This is in accordance with the theory presented above.

The index mapper in the WHT-OFDM-IM selects k subcarriers from n subcarriers as activated subcarriers for each subblock β , using the index bit p_1 . Then, the M-ary modulator maps the symbol bit p_2 to generate a vector of k modulation symbols. The combination of activated subcarrier indices can be expressed as:

$$S_{\beta} = \{s_{\beta}^1, s_{\beta}^2, \dots, s_{\beta}^k\} \quad (5)$$

TABLE I
INDEX MAPPING

p_1 bits	Subcarrier index combination	Subblock
0 0 0	2, 1, 0	[0 0 s(1) s(2) s(3)]
0 0 1	3, 1, 0	[0 s(1) 0 s(2) s(3)]
0 1 0	3, 2, 0	[0 s(1) s(2) 0 s(3)]
0 1 1	3, 2, 1	[0 s(1) s(2) s(3) 0]
1 0 0	4, 1, 0	[s(1) 0 0 s(2) s(3)]
1 0 1	4, 2, 0	[s(1) 0 s(2) 0 s(3)]
1 1 0	4, 2, 1	[s(1) 0 s(2) s(3) 0]
1 1 1	4, 3, 0	[s(1) s(2) 0 0 s(3)]

The index mapping method in WHT-OFDM-IM, like OFDM-IM, is comprised of two main approaches: **(1) the look-up table method** and **(2) the combinatorial method** [4].

(1) The look-up table method: Given the number of subcarriers, $n = 5$ and the number of activated subcarriers, $k = 3$, for example, since there are $C(5, 3) = 10$ possible permutations of active subcarriers, the proposed WHT-OFDM-IM scheme eliminates 2 of these permutations, as shown in Table I.

(2) Combinatorial method: it provides a method to map the natural number and the index position in the permutation $C(n, k)$ to address the problem that the look-up table is too large when the index bit is too large. Therefore, index mappings are typically computed using combinatorial methods.

Assuming that the number of activated subcarriers is k , the index positions of the k activated subcarriers arranged in descending order can be represented as $J = \{c_k, \dots, c_1\}$. For any positive integer $Z \in [0, C(n, k) - 1]$, there is a unique set of J , resulting in J and Z a one-to-one mapping in the combinatorial number method. For instance, when the number of activated subcarriers $k = 3$, the mapping relationship can be expressed as:

$$Z = C(c_3, 3) + C(c_2, 2) + C(c_1, 1) \quad (6)$$

At first, the p_1 bit of index is converted directly to a positive decimal integer Z in each WHT-OFDM-IM subblock at the transmitter. Then look for the largest c_k such that $C(c_k, k) \leq Z$ for the determined Z , and select the largest c_{k-1} such that $C(c_{k-1}, k-1) \leq z - c(c_k, k)$, and so on to determine all the index positions. Finally, the redundant activated subcarrier permutations are abandoned. For example, $n = 7, k = 3, C(7, 3) = 35$, it has been shown like this:

$$\begin{aligned} 34 &= C(6, 3) + C(5, 2) + C(4, 1) \rightarrow J = \{6, 5, 4\} \\ 33 &= C(6, 3) + C(5, 2) + C(3, 1) \rightarrow J = \{6, 5, 3\} \\ &\dots \\ 1 &= C(3, 3) + C(1, 2) + C(0, 1) \rightarrow J = \{3, 1, 0\} \\ 0 &= C(2, 3) + C(1, 2) + C(0, 1) \rightarrow J = \{2, 1, 0\} \end{aligned}$$

Based on the above theoretical analysis, the activated subcarriers are obtained by index mapping p_1 bit. For

the symbols on the activated subcarriers, traditional M-QAM/M-PSK modulation is carried out according to the information of p_2 bit. Thus, the WHT-OFDM-IM symbols can be expressed as:

$$X_\beta = \{x_\beta^1, x_\beta^2, \dots, x_\beta^n\} \quad (7)$$

Furthermore, in the WHT-OFDM-IM, we use the IFFT algorithm to complete orthogonal operation to highlight the essence as a novel OFDM system.

$$Y_\beta = \text{IFFT}\{X_\beta\} \quad (8)$$

Compared with conventional OFDM-IM wireless systems, the data in WHT-OFDM-IM undergoes an IFFT operation followed by WHT. At the receiver, the inverse of this transform, IWHT is applied.

WHT is a type of non-sinusoidal function transform that uses orthogonal rectangular functions as its primary functions and exhibits properties similar to those of Fourier functions. It is performed by multiplying the input signal with a WHT matrix in the transmitter, and the received signal is multiplied with the same WHT matrix at the receiver to recover the signal. The relationship between the input and output signals can be represented as:

$$Z_\beta = H_i Y_\beta \Leftrightarrow Y_\beta = H_i Z_\beta \quad (9)$$

where Y_β and Z_β are input and output signals respectively, H_i is WHT matrix, a precoding matrix, whose code length $i = 2^l (l \in \mathbf{Z}^+)$. Assuming that H is a matrix orthogonal composed of 1 and -1 in order to more clearly illustrate the characteristics of WHT matrix, then the block matrix can be expressed as:

$$\begin{pmatrix} H & H \\ H & -H \end{pmatrix} \quad (10)$$

thus, the general sequence of WHT matrix can be expressed as:

$$\begin{aligned} H_1 &= (1), H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \dots, \\ H_{i=2^l} &= \frac{1}{\sqrt{i}} \begin{pmatrix} H_{k=2^{l-1}} & H_{k=2^{l-1}} \\ H_{k=2^{l-1}} & -H_{k=2^{l-1}} \end{pmatrix} \end{aligned} \quad (11)$$

where $1/\sqrt{i}$ is expressed as a normalization factor, as the order of the Hadamard matrix, k is used to derive the WHT matrix.

Finally, we utilize Fast Walsh-Hadamard-Fourier Transform (FWHFT) algorithm to significantly reduce the computational and implementation costs of the WHT-OFDM-IM systems [13].

The task of the receiver in the WHT-OFDM-IM system is to detect the combination of activated subcarrier indices and the transmission symbols on the activated subcarriers. The most commonly used detection algorithms in conventional OFDM-IM systems, the Maximum Likelihood (ML) hard decision algorithm and the Log-Likelihood Ratio (LLR) soft decision detection algorithm, can also be applied in WHT-OFDM-IM systems [4]. However, in order to maximize the performance improvement of WHT-OFDM-IM compared to OFDM-IM, this paper adopts the ML detection algorithm.

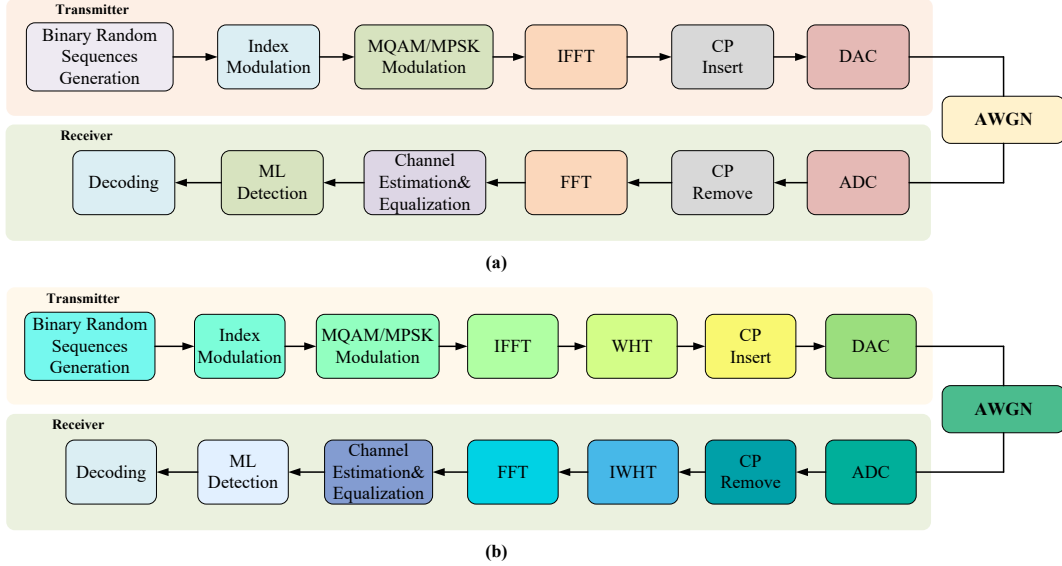


Fig. 2. Schematic of the OFDM-IM and WHT-OFDM-IM transceiver architecture: (a) is the OFDM-IM transceiver architecture, and (b) is the WHT-OFDM-IM transceiver architecture.

The core concept of the ML detection algorithm involves searching and comparing the combination of index \hat{I}^i and modulation symbol vectors \hat{s}^i for all possible activated subcarriers. The ML algorithm minimizes the value of the following expression:

$$(\hat{I}^i, \hat{s}^i) = \arg \min_{I^i, s^i} \sum_{a=1}^n |y_a^i - x_a^i|^2 \quad (12)$$

where \hat{I}^i is the activated index combination of each sub-block, and \hat{s}^i is the modulation symbol vector. $y^i(a)$ and $x^i(a)$ is the received signal and send signal respectively.

III. THE TRANSCEIVER STRUCTURE AND OPTIMAL PARAMETER OF WHT-OFDM-IM

The objective of this section is to outline the system architecture and establish the relevant simulation parameters in the WHT-OFDM-IM system, as depicted in Fig. 2. The architecture of the WHT-OFDM-IM transceiver is a combination of that of traditional WHT-OFDM and OFDM-IM transceivers [4], [9]. It integrates WHT between the IFFT and cyclic prefix in the transmitter and IWHT between the FFT and cyclic prefix in the receiver, as compared to the transceiver structure of OFDM and WHT-OFDM. However, the use of the WHT in signal processing will inevitably increase both the hardware and computational costs of the communication system. The functions performed by the transmitter include: index modulation, MQAM/MPSK coding, IFFT algorithm implementation, WHT algorithm implementation, adding of a cyclic prefix to the IFFT output, digital-to-analog (DAC) conversion, and finally transmission to the AWGN channel.

At the receiver, as depicted in Fig. 2, the received signal undergoes analog-to-digital (ADC) conversion, removal of the cyclic prefix, and processing via the IWHT algorithm. The output of the IWHT algorithm is then subjected to FFT. The channel estimation and equalization operation

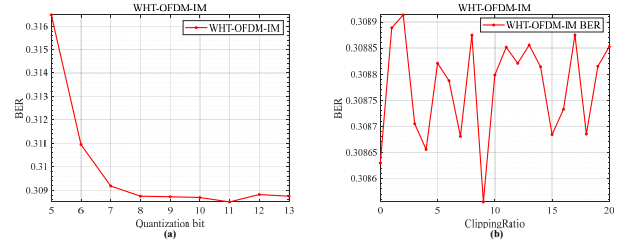


Fig. 3. Schematic illustration of optimal parameters: (a) is the optimal quantization bit in the WHT-OFDM-IM, and (b) is the optimal clipping ratio in the WHT-OFDM-IM.

is carried out at this stage to mitigate the impact of the channel on the ML detection algorithm. This paper adopts the ideal zero-forcing (ZF) equalization.

$$Y_E = \frac{P_p}{P_p \cdot P} Y \quad (13)$$

where Y is received signal by FFT, P is the estimated channel impulse response, P_p is the complex conjugate transpose of P .

The final step in the receiver process involves utilizing the Maximum Likelihood (ML) detection algorithm and the corresponding modulation format decoding. The optimal quantization bit and clipping ratio for the WHT-OFDM-IM system, when operating in an AWGN channel, are illustrated in Fig. 2. As shown in Fig. 3(a), the optimal quantization bit is 11 bits. If the quantization bit is less than 11 bits, the quantization error introduced by the ADC and DAC converters increases, resulting in a higher BER. Conversely, when the quantization bit is greater than 11 bits, the increased sensitivity of the ADC/DAC converters does not enhance system performance. Additionally, Fig. 3(b) reveals that the optimal clipping ratio is 9 dB, which has a minimal impact on the BER in the WHT-OFDM-IM system.

TABLE II
THE SIMULATION PARAMETERS SETTING

Parameters	Value
Subcarrier numbers	16
Activated subcarriers	12
Length of sequence	140000
Modulation formats	64QAM
CP	0.125
DAC & ADC quantization bit	11
clippingratio	9dB
Detection	ML
SNR	0~25dB
PAPR	0~12dB

TABLE III
THE PARP COMPARISON

Activated subcarriers	PAPR before WHT/dB	PAPR after WHT/dB	Performance
8	5.0081	5.6142	+12.10%
10	5.1007	5.2787	+1.03%
12	5.1616	5.0162	-2.82%
14	5.2042	4.8051	-7.67%

IV. SIMULATION RESULTS

Combined with the simulation of the optimal parameters and theoretical analysis in Section III, the optimal simulation parameters are obtained as shown in Table II. If the difference of parameters is not indicated in this section, all simulation parameters refer to Table II.

Table III provides a comparison of PAPR estimates between the OFDM-IM and WHT-OFDM-IM systems for different numbers of activated subcarriers. The results indicate that when the total number of subcarriers is equal to 16 and the number of activated subcarriers is equal to 8 or 10, the PAPR of the WHT-OFDM-IM system is higher than that of the OFDM-IM system. However, when the number of activated subcarriers is equal to 12 or 14, the PAPR of the WHT-OFDM-IM system is lower than that of the OFDM-IM system.

The CCDF curve comparing OFDM-IM with the WHT-OFDM-IM under different numembers of activated subcarriers when the number of subcarriers is 22 are shown in Fig. 4. The PAPR of the WHT-OFDM-IM system is lower than that of the OFDM-IM system in this scenario. Thus, WHT-OFDM-IM can combat the issue of PAPR in multiple subcarrier and multiple activated subcarrier scenarios.

The results demonstrate that the WHT-OFDM-IM can effectively reduce the high PAPR in scenarios with multiple activated subcarriers, which at least more than half of the total subcarriers, thereby overcoming the traditional OFDM-IM's drawback as an OFDM system. Additionally, as the number of activated subcarriers increases, the improvement in PAPR reduction becomes closer to that of WHT-OFDM. However, as the number of activated

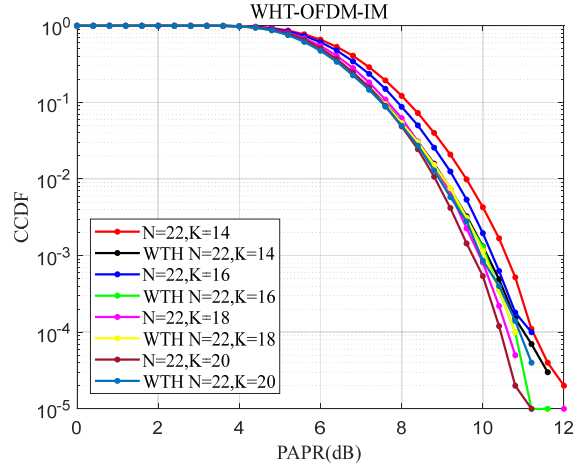


Fig. 4. CCDF curve comparing OFDM-IM with the WHT-OFDM-IM under different numembers of activated subcarriers.

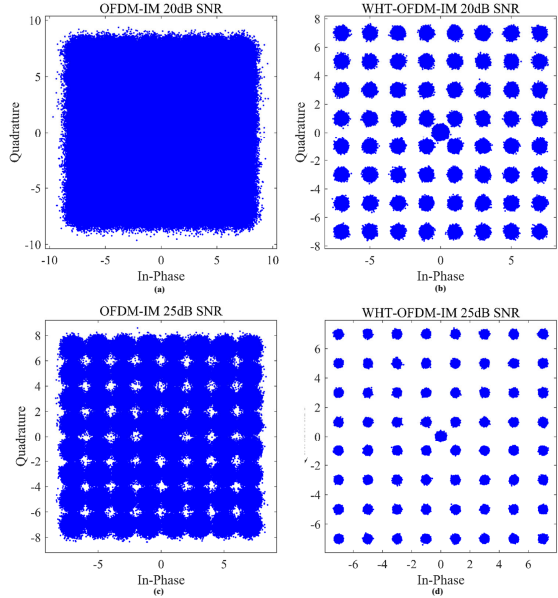


Fig. 5. Constellation of received signals between WHT-OFDM-IM and OFDM-IM under the same SNR: (a) is the constellation of received signals in OFDM-IM under 20dB SNR, (b) is the constellation of received signals in WHT-OFDM-IM under 20dB SNR, (c) is the constellation of received signals in OFDM-IM under 25dB SNR, and (d) is the constellation of received signals in WHT-OFDM-IM under 25dB SNR.

subcarriers increases, the signal bit rate R_b decreases, thus future research should focus on finding an optimal balance between PAPR and signal bit rate R_b in WHT-OFDM-IM.

In an AWGN channel, the impact of Gaussian white noise on the received signal is depicted in Fig. 5. This shows the constellation of received signals in WHT-OFDM-IM compared to OFDM-IM under 20 dB SNR, while Fig. 5(c) and Fig. 5(d) depict the same comparison under 25 dB SNR. As shown in Fig. 5, WHT-OFDM-IM signals are less affected by noise than OFDM-IM, and the distinguishability of received signal constellation points is significantly enhanced by WHT.

Moreover, the BER and SER performance of WHT-OFDM-IM and OFDM-IM versus SNR are presented in

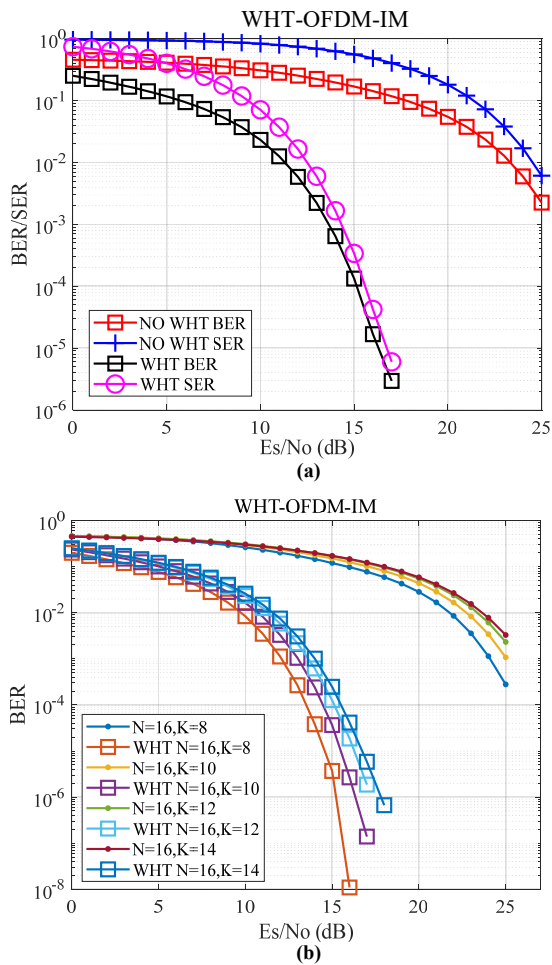


Fig. 6. Simulation of the BER and SER between OFDM-IM and WHT-OFDM-IM: (a) is the BER and SER simulation comparing OFDM-IM with the WHT-OFDM-IM, and (b) is the BER and SER simulation comparing OFDM-IM with the WHT-OFDM-IM under different numbers of activated subcarriers.

Fig. 6. The comparison is depicted in Fig. 6(a), which shows that WHT-OFDM-IM has superior transmission performance as WHT significantly reduces the SER and BER in OFDM-IM. Compared to traditional OFDM-IM, WHT-OFDM-IM incurs approximately 11 dB SNR penalty when the BER is 1.0×10^{-3} . In line with the theoretical results reported in [11], WHT not only improves the transmission performance of OFDM, but also enhances that of OFDM-IM.

As shown in Fig. 6(b), the effect of the number of activated subcarriers on the BER and SER performance was evaluated by applying different numbers of activated subcarriers. It can be observed that the addition of WHT in OFDM-IM does not impact the BER and SER performance with regards to the number of activated subcarriers. However, as the number of activated subcarriers increases, both the SER and BER will increase, which is in line with the inherent properties of OFDM-IM. This demonstrates that WHT does not alter the fundamental characteristics of OFDM-IM, and only enhances its transmission performance through the addition of simple DSP components for performing WHT and IWHT at the transmitter and

receiver, respectively. This intriguing finding aligns with the demands of next-generation wireless networks [14].

V. CONCLUSION

In conclusion, this paper presents the WHT-OFDM-IM system, which integrates the WHT and OFDM-IM techniques to improve transmission performance and effectively combat the PAPR problem in OFDM-IM. Simulation results demonstrate the high efficiency and PAPR friendliness of the proposed system for a large number of activated subcarriers, making it a promising candidate for large-scale subcarrier systems. Furthermore, the WHT-OFDM-IM system is capable of mitigating the cost of HPA associated with the high PAPR of OFDM-IM signals when passing through non-linear devices. Future research will focus on finding an adaptive balance between PAPR and signal bit rate R_b in WHT-OFDM-IM. Additionally, the WHT-OFDM-IM system exhibits significantly reduced BER and SER compared to traditional OFDM-IM, making it a highly attractive solution for the next generation of wireless communication systems.

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